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GRADE 12 LEARNERS' UNDERSTANDING AND MISCONCEPTIONS WHEN LEARNING FUNCTIONS AND DERIVATIVES GRAPHS IN CALCULUS

Abstract. Calculus is based on an understanding of the graphical depiction of the derivatives and its functions which remain important in mathematics, yet there are many misconceptions which could impede a student's advancement in these topics. On this note, the idea that the slope of the line passing through a point represents the derivative of a function at that point is a common fallacy. Because this idea ignores the believe that the tangent line is the limit of secant lines as they approach the point of tangency. Therefore, the ideas of the derivatives and their uses were seen in an overly simplistic light. These misinterpreting of the derivative as a ratio of dy to dx instead of seeing it as the average rate of change's limit is another prevailing misunderstanding among the students. Due to this misperception, differentiation was approached mechanically without consideration for its theoretical foundations among the students. As a result of this, some students find it difficult and often misuse it, misidentifying the inner and outer functions and failing to recognize the composition of functions. And this had led to certain misunderstandings and misconceptions when it came to applying the graphical method for computing derivatives in mathematics. The result of this study had led to recognizing some misconceptions, including incorrect assumptions made when learning new concepts, and this include, conceptual misconceptions, conceptual errors, generalization or transfer errors, ignorance of rule restrictions, and insufficient application of rules. A qualitative method was used for the study with the selection of Grade 12 math students from a South African high school to accomplish the study's goal.

Keywords: misconception; knowledge; calculus; function; graph.

Introduction

The teaching of calculus among Grade 12 students ought to be done with a basic understanding of functions and derivative graphs, and how to represent them graphically, but some studies had reported some misconception commonly found among grade 12 students (Jameson et al., 2024; Syarifuddin & Sari, 2021). This is due to some factors which include, a lack of conceptual clarity, inadequate teaching strategies, and incorrect use of graphical symbols (Bursa & Polar, 2020; Jameson et al., 2024; Ozaltun-Celik et al., 2021). Due to this, some students are struggling with the abstract nature of functions taught among the students, which had led to a superficial understanding that hinders their ability to apply concepts to new problems. For instance, the misconception that a function's graph can be drawn without considering its domain and range is common. This leads to errors in sketching accurate graphs, which are essential in visualizing the behavior of functions (Bragdon et al., 2019). Moreover, some students don't fully grasp the significance of key features such as asymptotes, intercepts, and intervals of increase and decrease, which are vital in interpreting real-world scenarios (Hadjidemetriou & Williams, 2002)**.** To mitigate these issues, there is a need of emphasizing on the deeper conceptual understanding through interactive learning experiences, real-life applications, the use of technology to visualize functions dynamically and avoid overgeneralization (Fasinu, 2024). By addressing these misconceptions, we must enhance students' comprehension and enable them to harness the power of calculus as a tool for solving complex problems (Atar & Aykutlu, 2023).

Certainly, there are some common misconceptions in calculus, particularly in derivative graph which often stem from foundational concepts being misunderstood (Syarifuddin & Sari, 2021). And some of these include: One, the misconceptions about the belief that a continuous function must be differentiable everywhere, which is not the case. Two, the misconception about the confusion between the point of inflection and the concavity of a function which allow the students to sometimes mistakenly believe that a point of inflection implies a change in concavity (Körner, 2005). Additionally, there's the misconception regarding the limit of a function, where students may assume that if a function approaches a value as x approaches infinity, the function must equal to that value at infinity. Another is a form of a misunderstanding that the true meaning of an asymptote is also prevalent; it is often thought to be a line that a graph cannot cross, whereas, in reality, a graph can cross its asymptotes. Lastly, the improper use of notation and terminology, such as confusing the delta (Δ) symbol with the derivative (d), can lead to significant errors in understanding and applying calculus principles. Addressing these misconceptions requires careful instruction and a focus on conceptual understanding to ensure students develop a robust mathematical foundation (Ozaltum-Celik, 2021; Syarifuddin & Sari, 2021; Körner, 2005).

However, some students can self-correct their misconceptions in calculus by actively engaging in reflective practices and seeking a deeper understanding of the concepts (Körner, 2005). One effective strategy is to regularly review and analyze solved problems, identifying any errors in reasoning or application. This process can be enhanced by discussing problems with peers or educators, which often leads to new insights and corrections of misunderstandings. Additionally, utilizing technological tools such as graphing software can help students visualize functions and their properties, making abstract concepts more concrete. It's also beneficial for students to approach problems from multiple perspectives, as this can reveal underlying misconceptions and provide a more rounded comprehension of the material. Regularly testing one's understanding through practice problems that challenge common misconceptions can further solidify correct knowledge. Finally, accessing resources that specifically address misconceptions in calculus can provide targeted strategies for overcoming these hurdles (Ozaltum-Celik, 2021).

The concept of higher-order derivatives also poses challenges, with students sometimes viewing them as merely repeated applications of the first derivative, rather than as successive derivatives that provide deeper insights into a function's behavior, such as concavity and points of inflection. There is also a tendency to overlook the geometric interpretation of the second derivative as the curvature of the graph. Students may also harbor the misconception that all functions are differentiable everywhere, which is not the case. Functions can have points or intervals where they are not differentiable, such as corners, cusps, or vertical tangents. This misunderstanding can lead to incorrect assumptions about the existence of derivatives and their properties Jameson et al., 2024). The derivative of a constant function being zero is another concept that is often misunderstood. Some students mistakenly believe that since a constant function has no rate of change, it should not have a derivative at all. This reflects a lack of understanding that the derivative quantifies the rate of change, and a zero derivative accurately represents the constant function's static nature. Furthermore, when dealing with implicit differentiation, students might not fully comprehend that they are differentiating both sides of an equation with respect to x, which can lead to errors in applying the derivative rules. This is compounded by the confusion between differentiating with respect to a variable and taking the derivative of an expression (Jameson et al., 2025). In the context of applied problems, such as related rates or optimization, students may not correctly translate the real-world situation into a mathematical model involving derivatives. This can result in the misapplication of differentiation techniques and incorrect solutions to problems (Zehra & Abbasi, 2019). To combat these misconceptions, educators must emphasize a conceptual understanding of derivatives, linking the algebraic processes to geometric interpretations and real-world applications. Interactive visual tools and carefully designed problem sets can help students build a robust understanding of derivatives, their properties, and their significance in calculus. By addressing these common misconceptions, students can develop a more nuanced and accurate understanding of derivatives, paving the way for success in calculus and beyond (Jameson et al., 2025). It is through this deeper comprehension that students can appreciate the beauty and utility of derivatives in mathematics.

This study was guide by the research question stated below.

- What are the common Knowledge and misconceptions possess by the Grade 12 students when learning functions and derivatives graphs in Calculus?

In achieving the goal of this study, the researchers discussed the views of other researchers and methodology adopted in carrying out this study.

Mathematics reasoning and misconception on Graphical representations

The journey through calculus is often a pivotal moment for Grade 12 students, as it represents a significant leap into the abstract and complex world of advanced mathematics. A fundamental aspect of this journey is the understanding of functions and their graphical representations. Knowledge in this area is crucial, as it forms the backbone of calculus and its myriad applications. However, misconceptions can arise, sometimes from a lack of connection between the algebraic and geometric perspectives of functions. Students may grasp the procedural mechanics of calculus without truly understanding the underlying concepts. For instance, they might be able to compute a derivative but fail to recognize its graphical interpretation as the slope of a tangent line at a point on a curve. This disconnect can lead to a superficial understanding of calculus, where students are equipped to solve equations but are ill-prepared to apply these principles to real-world problems.

Another common misconception is the belief that a function's graph can be sketched by plotting a few points and connecting them with straight lines. This oversimplification ignores the nuances of continuity and differentiability, essential concepts that influence the shape and behaviour of graphs. Students must appreciate that functions can exhibit a wide range of behaviours, such as asymptotes, discontinuities, and intervals of increase or decrease, which are critical to understanding the nature of the function (Rodriguez & Jones, 2021). Moreover, the misinterpretation of the domain and range of functions can lead to erroneous conclusions about their graphs. Students often struggle with the concept of domain, particularly with functions involving roots and fractions, which can lead to incorrect graphs that do not accurately reflect the function's properties. Similarly, confusion about the range can result in graphs that either overextend or truncate the true scope of the function's values. To address these misconceptions, educators must emphasize a conceptual understanding of functions and their graphical representations. This involves linking algebraic expressions to their visual counterparts and encouraging students to analyses the behaviour of functions beyond mere point plotting. Interactive tools and software can aid in this visual learning, allowing students to experiment with functions and observe the immediate impact on their graphs.

In conclusion, while Grade 12 students may face challenges in mastering functions and graphical representations in calculus, these obstacles can be overcome with a strong conceptual foundation. By dispelling misconceptions and fostering a deeper comprehension of the relationship between functions and their graphs, students can unlock the full potential of calculus as a powerful tool for analysis and problem-solving (Darren, 2019). As they transition from high school to higher education, this knowledge will serve as a critical asset in their mathematical toolkit. Calculus, that intricate dance between functions and their rates of change, captivates both novices and seasoned mathematicians. As students grapple with its nuances, they often encounter misconception on those elusive shadows that can lead them astray (Makonye & Luneta, 2016; Ozaltun-Celik, 2021). In this exploration, we'll delve into the mental landscape of calculus learners, shedding light on their understanding of graphical approaches to derivatives (Macey, 2019; Ozaltun-Celik, 2021). To teach derivatives effectively, we must guide students toward visualizing the concept. Graphs become our canvas, allowing us to paint the essence of slopes, tangents, and instantaneous change (Zehra & Abbasi, 2019).

Methods

The section present the method used by the researchers in collecting and reporting their data on the common knowledge and misconceptions on Common Knowledge and Misconceptions Among Grade 12 students when learning Functions and Derivatives Graphs in Calculus. The Grade 12 mathematics students from one of the province's secondary schools were chosen for this study, which used a practical and purposeful sampling technique. Reaching the students was simple and convenient because one of the researchers worked as an educator at the college. Differential calculus is covered in the grade 12 mathematics curriculum; hence the chosen sample included the necessary data.

Research Design. This is a logical method adopted by the researchers when collecting a data for a study and its required some major stages which include, sample and sampling, stages of data collection and researcher participants among many others. To validate the accuracy of this study, the process adopted for data collection and analysis are carefully explained below.

Participants. For this study, 35 Grade 12 students were invited after the the consent form had been completed, and their views on how they calculate some aspects of derivative and the functions graph were tested. Their answers to the survey questionnaires assisted the researchers to recognise their understanding on the functions, graph and the approaches of drawing them. Therefore, seven students were chosen utilizing their worksheets based on the data gathered from thirty-five Grade 12 students. Five students were then chosen for the interview section. Each of these students made some valuable contributions on their understanding of the misconceptions that 12th graders have when learning calculus, functions, and derivative graph. All these were done with the assistance of qualitative method to meet up with the objective of the study.

Conceptual Framework

Previous framework on the avoidable misconceptions on the calculus (i.e. derivatives and functions graphs) as taught among grade 12 students in south Africa had reported some researchers among which are Jameson et al. (2025), Jameson et al. (2024), Makonye and Luneta, (2016), Tohl (2021) and Ngulube and Ogbonnaya (2023). Furthermore, Asiala et al. (1997), Borji et al. (2018), Delos-Santos and Thomas, (2005), Jones and Watson (2018), Özgen, et al. (2017), and Park et al. (2013). But Zandieh (2000) and others explained the developmental progressions that are being discovered when teaching the process of the derivative and functions as topic in Grade 12 level. In his study, Zandieh reported a framework for exploring students' understanding of derivative which was said to contains four contextual ideas and these include; the graphical context, verbal context, paradigmatic context and a symbolic context.

Accordingly, Borji and others who reported that when students are studying derivaties and function using a graphical illustration, they find the slope of a secant line through two points on the graph. Secondly, they use the concept of limit to find the slope of the tangent line at a point by thinking of approximation points on the curve to a specific point. Finally, comprehending the derivative as a function requires understanding that the slope is different for different values of the independent variable (Borji et al., 2018b). Therefore, when learning some complex mathematical ideas on graph, some misconception could be classified into four division as suggested by on domain, function, sketching method and plotting procedure (Syarifuddin & Sari, 2021).

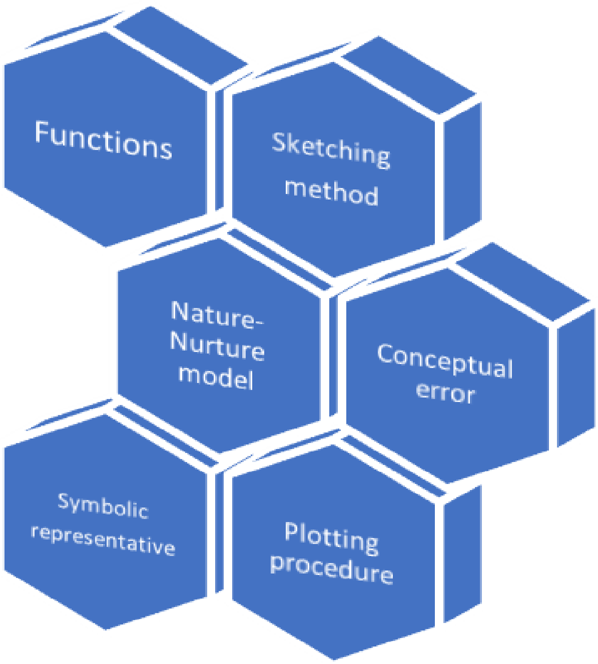


Figure 1: **Nature, Nurture-based Graphical misconception**

Gathering from the framework stated above, it could be said that some of the misconceptions among the students when learning functions and derivatives could be discovered when the learners are calculating some important concepts like functions, derivative and the rate of change before transforming them into a graph. For instance, if a students understand derivative, he/she could be able to explain the average range of change which form how the slopes of secant lines and the slope of tangent line could be reported But the fact remains that during the learning of these concepts some students run into some errors as a result of an inadequate understanding on how to go about them, and some of these misconception include; Conceptual misconception, generalisation or transfer errors, ignorance of the function rule, incomplete application of the sketching method, wrong hypothesis and plotting procedure among. All these came in due to low instrumental understanding of the derivative concept (Sahin et al., 2015).

Data presentation

This section presents a data that reported the views of the participants on the common knowledge and Misconceptions among Grade 12 students when learning functions and derivatives graphs in calculus. In reporting the views of the participants, the following sections were carefully presented thematically using the views from the participants. And these were discussed using some categories and subcategories in the following sections.

Interview on learners understanding of f(x), $f^{/}(x)$ and $f^{//}(x)$

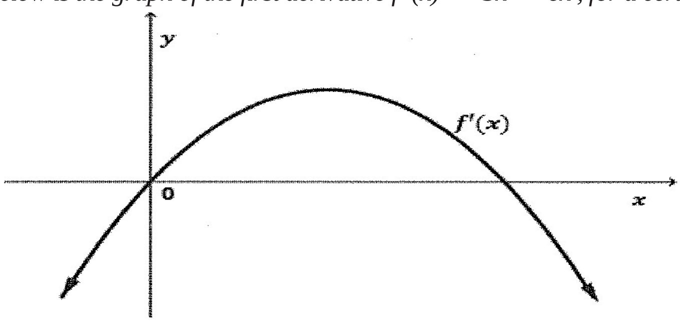
- Learner 8 misconceptions on $f(x), f^{/}(x)$ and $f^{//}(x)$
- Learner 9 misconceptions on $f(x), f^{\square}(x)$ and $f^{\square}(x)$
- Learner 10 misconceptions on $f(x), f^{/}(x)$ and $f^{//}(x)$
- Learner 11 misconceptions on $f(x), f^{/}(x)$ and $f^{//}(x)$
- Learner 12 misconceptions on $f(x), f^{/}(x)$ and $f^{//}(x)$

All these were reported to described the views of the grade 12 learners on their common knowledge and misconceptions on function and derivative graph when learning calculus. On this note, the thematic section reported below, was anchored the sample question shown as followed.

Sample question 1for testing grade 12 Learners' misconceptions on $f(x), f^{/}(x)$ and $f^{//}(x)$

For this study, the question below was used to investigate the common knowledge and misconceptions among the Grade 12 students when learning functions and derivatives graph in calculus. And a careful analysis was reported in the succeeding sections.

Question 1. *Below is the graph of the first derivative $f^{/}(x) = -3x^2 + 6x$, for a certain function f(x)*



1.1 Calculate the x -coordinates of the stationary points of f(x). Justify your method.

1.2 Determine value of x at the point where the concavity of f(x) changes. Use given graph to explain your answer.

1.3 It is further given that $f(x) = -ax^3 + bx^2 + cx - 4$. and f (2) = 0, draw a rough sketch of f(x), clearly showing the coordinates of the turning points. Give reasons to justify the sketch you have drawn.

Figure 2: **Sample question 1 for testing grade 12 Learners' misconceptions**

Question 1 aimed at assessing the learners' knowledge of functions and the interrelationship between functions and derivatives and the geometric meanings of the first and second derivatives. The focus was on whether learners could use the given graph of $f^{/}(x) = -3x^2 + 6x$ to make deductions on either *f(x) or* $f^{//}(x)$.

This was a higher order question to check on learners' understanding of $f^{/}(x)$ and how it relates to *f(x)* and $f^{//}(x)$. A few learners in this group of nineteen learners, (ci1) (19), could not explain why $f^{/}(x) = 0$ is used to find stationary points even though they found the correct answers. Others, on the other hand, treated $f'(x)$ as $f(x)$ and eventually worked with $f''(x) = 0$ to try and find stationary points, i.e. lacking understanding of the relationship between the three $f(x)$, $f'(x)$ and $f''(x)$. For example, L27 presented the following solution in response to question on finding turning points $f'(x) = -3x^2 + 6x$; $f(x) = -6x + 6 = 0$; $x = 1$, as shown in example below.

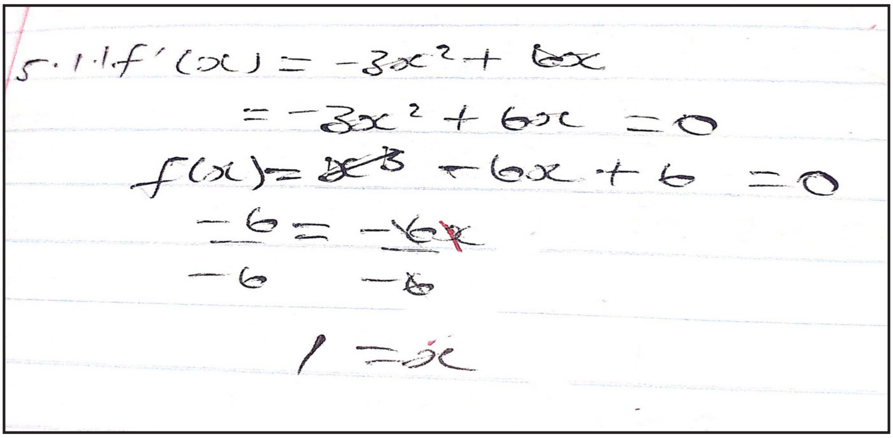


Figure 3: **L27's solution to question 5.1**

In Figure 3, L27 finds the derivative of $f'(x)$ but writes $f(x)$ when in fact he is now working with $f''(x)=0$ to address the question on turning points of $f(x)$. This type of error could be a product of learning which is dependent on drill where the learner has been conditioned to assume that to find a turning point, one must first find the derivative of the given function. In this case when L27 is presented with the derivative $f'(x)$, he fails to correctly resolve the calculus question because there is a lack of conceptual understanding of the derivative and *how it relates to the original function f(x). One learner, L16*, in this group ,(ci5) (19), introduced $f(x)$ from $f'(x) = -3x^2 + 6x$, giving $f(x) = -x^3 + 3x^2$ as shown in Figure 4.34 below.

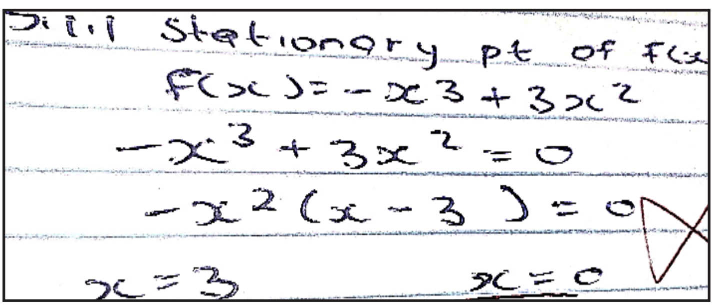


Figure 4: **L16's solution to question 5.1**

L16 failed to realize that $f'(x) = -3x^2 + 6x$ could be the derivative of any function $f(x) = -x^3 + 3x^2 + c$,where c is any constant. *L16* uses $f(x) = -x^3 + 3x^2 = 0$ in his attempt to address the question of turning points of $f(x)$. This is an indication that the learner has a shallow understanding of $f(x)$ and $f'(x)$ and how they relate to each other as it is only the first derivative $f'(x)$, which must be equated to zero to find turning points of $f(x)$.

Interview on learners understanding of f(x), f'(x) and f''(x)

Five learners *L8, L9, L10, L11* and *L12* were interviewed so that they clarify their written responses to question 1.

Learner 8 misconceptions on $f(x), f^{/}(x)$ and $f^{//}(x)$

The researchers probed the learner 8 to confirmed how the graph could be used to find the turning point. On this note the interaction had by the researchers and the students goes thus;

R: How do you use the graph of y= f $^{/}$*(x) to find the turning points of f(x).*

L8: The x-intercepts where y=0 show the turning points of f(x)

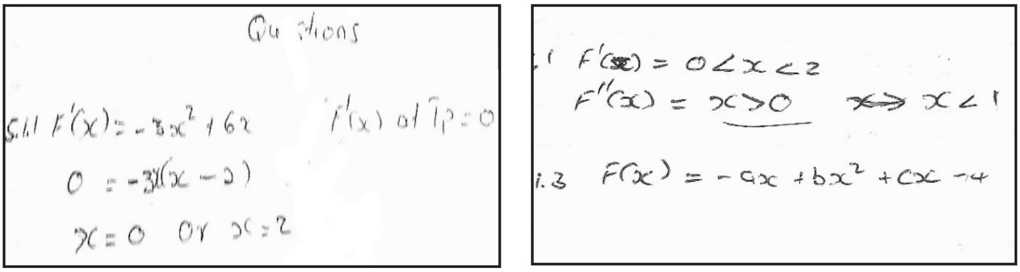


Figure 5: **L8's written responses to Question 1**

R;. Can you explain your answer on where the concavity of f(x) changes.

L8: Concavity of f(x) must change between x=0 and x=2. Using f $^{//}$*(x)= -6x+6 =0*

Concavity changes between $x > 0$ *and x*< 1.

R: Why did you not draw the graph of f(x) = - ax ^{3} *+ bx* ^{2} *+ cx – 4.*

L8: We are not given enough information to draw graph. The values of a, b and c must be known first.

L8 understood how to use graph of first derivative to identify turning points of *f(x),* but had misconceptions on how the second derivative is used to determine concavity of *f(x). L8* misconception was viewing concavity as changing over an interval when concavity changes.

at the point of inflection where *f* $^{//}$*(x)=-6x+6 =0.*

L8 lacked the relational understanding of *f(x) , f* $^{/}$ *(x) and f* $^{//}$*(x)* , as he failed to use turning point of graph of *f* $^{/}$ *(x)* to answer question on concavity changing at *f* $^{//}$ *(x)=0.* Secondly the learner failed to use answers from 1.1 on turning points of *f (x)* to sketch graph arguing there is insufficient information. This is evidence that the learner was treating each of these three *f(x) , f* $^{/}$ *(x) and f* $^{//}$*(x)* as separate concepts and not an interrelated structure of mathematical concepts.

Learner 9's misconceptions on $f(x), f^{/}(x)$ and $f^{//}(x)$

This section reports the result from the students on the common knowledge and misconceptions on the teaching and learning of functions and derivatives graph.

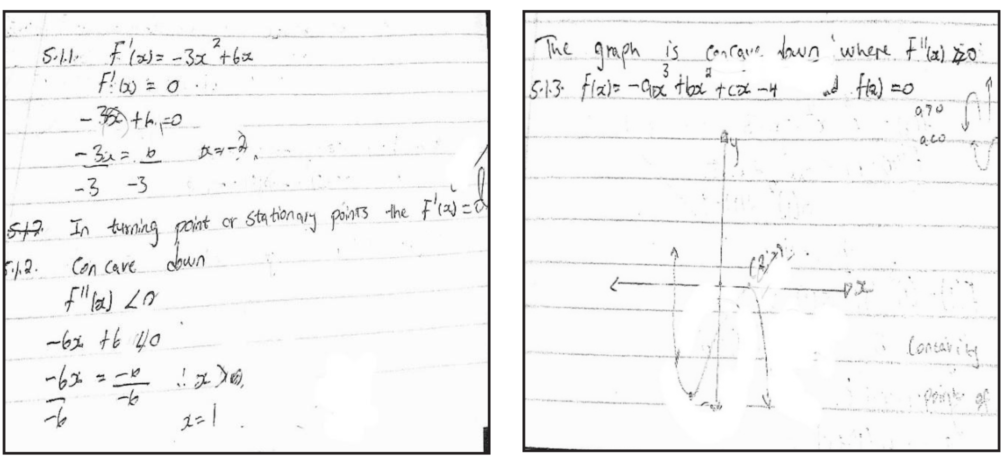


Figure 6: **L9 written responses to Question 1**

Researcher: How do you use the graph of y= $f^{/}(x)$ to find the turning points of f(x)?.
L9: At turning points $f^{/}(x)=0$, so we solve equation $-3x^{2}+6x=0$
Researcher Can you explain how you solved the quadratic equation $-3x^{2}+6x=0$ in 5.1
L9: You divide all terms by common factor x to get -3x+6=0. Then divide by -3 to get x= -2.
Researcher: Why did you write concave down when question asked for point where concavity changes?
L9: For concave down we find $f^{//}(x) < 0$, but when we solve $-6x+6 < 0$, x=1 which is bigger than zero which means graph is concave down when $f^{//}(x) > 0$.
Researcher: Can you explain which factors helped you to draw graph of $f(x) = -ax^{3} + bx^{2} + cx - 4$ in 5.3
L9: I used f(2)=0 ,so when x=2 ,y=0 and the coefficient of x^3 which is - a to get shape of graph. The other turning point must be at x= -2 but y value cannot be found.
L9's misconceptions were firstly on the solution of equations. In solving the equation $-3x^{2}+6x=0$, the learner divides all terms by an unknown value x, which is incorrect because he loses one of the solutions of the equation, that is x=0. Hence dividing by x, L9 is dividing by zero which mathematically is not allowed. This is not a misconception on calculus related concepts but is an error resulting from flaws in the mastery of extrinsic calculus concepts relating to the solution of algebraic solutions.
Secondly, L9 had a misconception on what is meant by "concavity changing". The concept of concavity had been misunderstood as the learner introduces "concave down" in place of "concavity changing". The first one is an interval while the second is a point of inflection, hence this is an indication that the learner is relying on memorisation of procedures, rather than on a conceptual understanding of the concepts. When the learner got an answer of x=1, there appeared to be confusion in his explanation that the answer is greater than zero, as if to imply he expected it to be less than zero from $f^{//}(x) < 0$. The learner's last explanation for "concave down" changes to "graph is concave down" where $f^{//}(x) > 0$", an indication that the learner was struggling to distinguish the values of the second

Learner 10's misconceptions on $f(x), f^{/}(x)$ and $f^{//}(x)$

This section reports the result from the students on the common knowledge and misconceptions among grade 12 learners when learning functions and derivative graph. And the report of the interactive section is hereby reported below.
Figure 7 below report all the participant understand about function and the derivative graph. misconceptions on the teaching and learning of functions and derivatives graph derivative, $f^{//}(x)$, from the values of x, and that there is no contradiction if one gets an answer of $x > 0$ when solving for $f^{//}(x) < 0$.
On the question of sketching the graph, L9 failed to establish the linkages between f(x) and the first derivative $f^{/}(x)$. The choice of f(2)=0 as a turning point appeared to be accidental as it is not supported by any correct working from $f^{/}(x) = -3x^{2} + 6x = 0$. The graph of $f^{/}(x)$ is given showing two x-intercepts which were not utilised by the learner in sketching the graph of f(x). This is evidence that the learner lacked the relational understanding of the function, f(x), and its first derivative, $f^{/}(x)$, as well as what information is conveyed by each graph. The learner fails to identify the y-intercept of the graph in $f(x) = -ax^{3} + bx^{2} + cx - 4$.
These misconceptions for L9 resulted from the learner failing to understand intrinsic calculus concepts on the relationship between the function, *f(x)*, and its derivative and the geometric meaning of the second derivative.

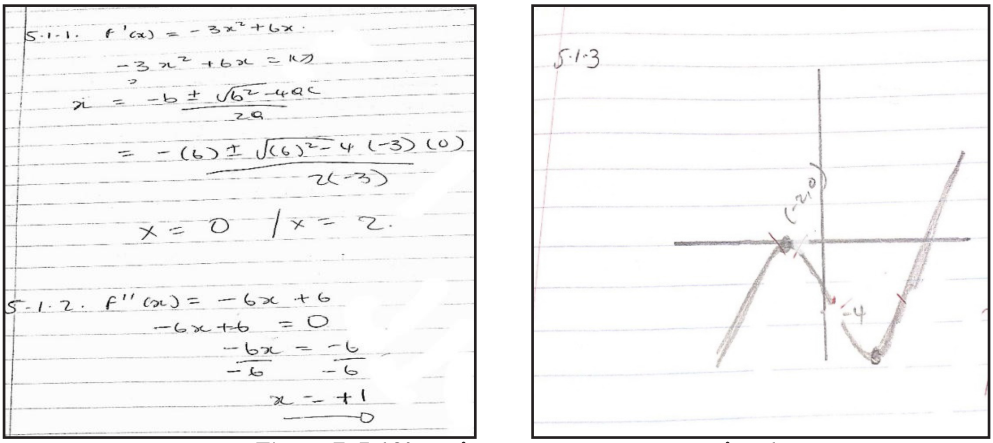


Figure 7: **L10's written responses to question 1**

Researcher *:Give a reason for your method of finding stationary points of f(x).*
L10:*For a stationary point, the first derivative is zero , so* $f^{/}(x) = -3x^2 + 6x$ *is equal to zero.*
Researcher: *How do you use the graph of y=* $f^{/}(x)$ *to find the turning points of f(x).*
L10:*If y=* $f^{/}(x)$*, then y=0, it is at the x-intercepts of graph.*
Researcher: *How can we find point of inflection from the given graph of* $f^{/}(x)$*?*
L10: *To find point of inflection we use* $f^{//}(x)=0$*, not the graph of* $f^{/}(x)$*. We find the second derivative and equate to zero.*
Researcher*: When drawing graph of f(x) you marked a point y= -4 on the y-axis, what does it mean ?*
L10:*I used x=0 for y-intercept of graph of* $f(x) = -ax^3 + bx^2 + cx - 4$
Researcher: Why do you have a turning point at (-2;0)?
L10*: I used f (2) = 0 , but when I use (2;0) the shape of cubic graph will not be correct*.
Researcher: Can you explain the answers you got in 5.1 of x=0 / x=2 . What were you calculating?
L10: At turning point $f^{/}(x) = 0$, so we solve $-3x^2 + 6x = 0$
Researcher:Why did you not use your answers for 5.1 to answer 5.3 to sketch graph of f(x)?
L10:Question 5.1 is for $f^{/}(x) = -3x^2 + 6x$ and 5.3 is for $f(x) = -ax^3 + bx^2 + cx - 4$, they are different. Here f(x) has many unknowns.
L10's insistence on using $f^{//}(x)=0$,is a reflection shows that he was relying more on memorisation of procedures for finding the point of inflection and did not understand that the turning point of the graph of $f^{/}(x)$ the gives point where concavity changes and it is symmetrical about the two turning points of f(x), that is x=0 and x=2. L10 evidently lacked a relational understanding of f(x), $f^{/}(x)$ and $f^{//}(x)$.The graph that L10 drew is additional evidence of the learner's shallow understanding of the concepts. He did not seem to understand the meaning of f (2)= 0, as it is not marked on the graph and is not used in addressing the problem.
Secondly, the learner failed to utilize $f^{/}(x) = -3x^2 + 6x = 0$ in determining the turning points of the required graph, even after correctly solving for x-coordinates and identifying and marking the y-intercept of *f(x)*. L10 appeared to address questions on a part by part basis and thus failed to see the interrelationship between the concepts *f(x)*, $f^{/}(x)$ and $f^{//}(x)$ as he could solve $f^{/}(x) = 0$ in *5, but* did not seem to appreciate how this relates to the graph of *f(x)*in *5.3*. The learner's approach to resolving given problems reflects an instrumental understanding of the calculus concepts as opposed to a relational understanding.

Learner 11's misconceptions on $f(x), f^{/}(x)$ and $f^{//}(x)$

This section reports the result from the students on the common knowledge and misconceptions on the teaching and learning of functions and derivatives graph

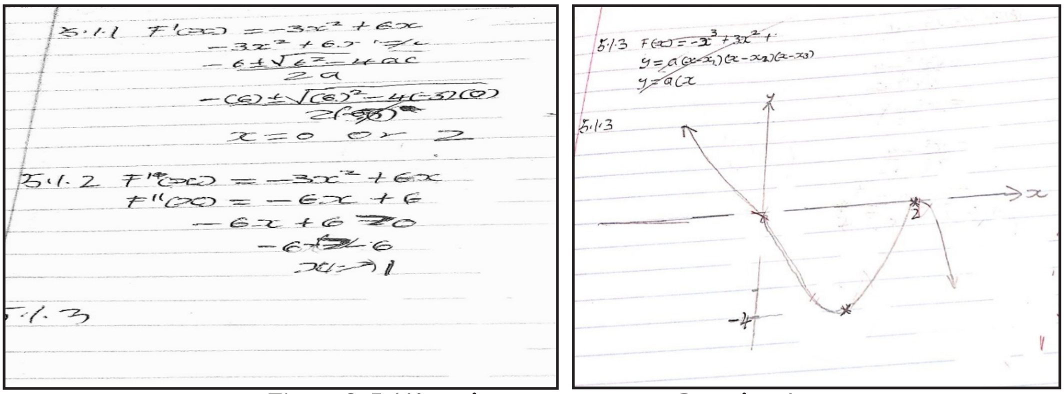


Figure 8: **L11's written responses to Question 1**

Researcher**:** *How do you use the graph of y= $f^{/}(x)$ to find the turning points of f(x).*
L11: *To find turning points we put $f^{/}(x)$=0 and solve.*
Researcher: *Explain your answer on concavity changing in 5.2*
L11: *When concavity is changing the second derivative is greater than zero.*
Researcher: *In your graph in 5.3, you marked a point y= -4 on the y-axis, what does it mean?*
L11: Y-intercept, x=0, so when I substitute x=0 in f(x) = $- ax^{3} + bx^{2} + cx - 4$ answer is – 4.
Researcher: Your graph shows x-intercepts x=0 and x=2, where did you get them?
L11: If we solve $- 3x^{2} + 6x = 0$, answer is x=0 or x=2 when y=0 at x-intercept
Researcher: Why do you have a minimum turning point between x=0 and x=2
L1I: I used the turning point of the derivative; it is between x=0 and x=2

The first misconception for L11 was that concavity changes when $f^{//}(x) > 0$. The concept has been misunderstood as the point of inflection is the point where concavity changes, and it is not an interval. The second misconception was on the solution of inequalities. L11 gave the answer $x > 1$ from -6*x* > -6, yet the correct solution should be $x < 1$ after the division with -6 on both sides. This error is a result of a weak background on extrinsic calculus concepts relating to the solution of equations and inequalities, which were learnt in earlier grades.

The graph and responses of L11 to oral questions reflected that even when the learner was able to mark the y-intercept, i.e. $y = -4$ when $x = 0$, he fails to use the point to sketch the graph of *f(x)* = $- ax^{3} + bx^{2} + cx - 4$, reflecting a lack of understanding of the fact that $f(0) = -4$ is a point on the graph. Secondly the learner failed to link $f^{/}(x) = - 3x^{2} + 6x = 0$ and $f(0) = -4$ to establish that f(0) = - 4 is another turning point for the graph, instead the minimum point on his graph seemed to be obtained by guesswork. The other misconception is that the learner uses the solutions of $f^{/}(x) = - 3x^{2} + 6x = 0$ for x-intercepts, yet this is a calculation for x -coordinates of turning points, so the x -intercepts of the graph of the first derivative are not necessarily the x -intercepts of the function *f(x).* The sources of these misconceptions are intrinsic calculus concepts of derivatives of functions and how to interpret the graph of the first derivative, in relation to the original function and the second derivative.

Learner 12's misconceptions on $f(x), f^{/}(x)$ and $f^{//}(x)$

This section reports the result from the students on the common knowledge and misconceptions on the teaching and learning of functions and derivatives graph

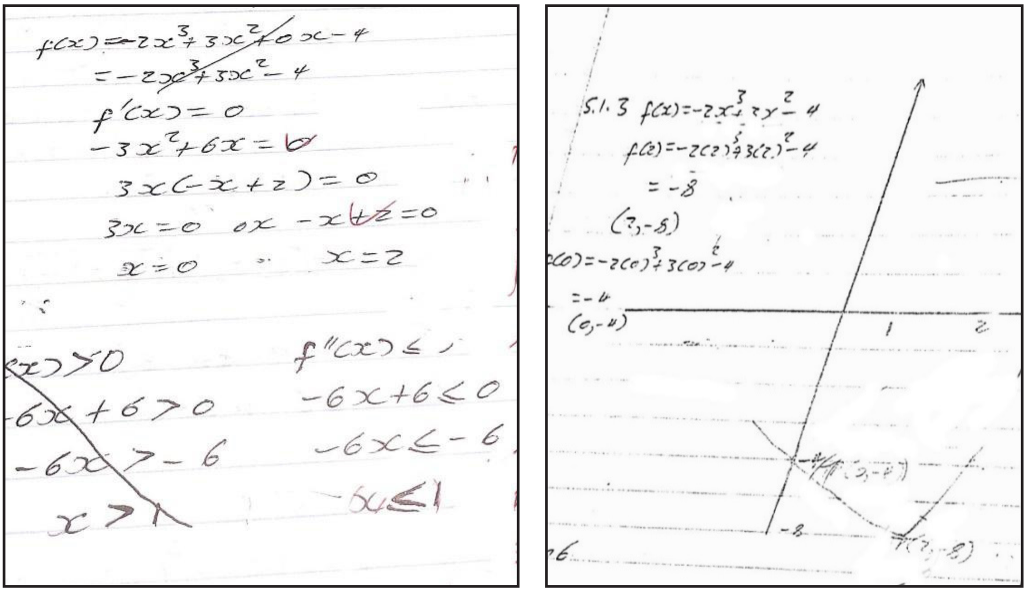


Figure 9: **L12's written responses to Question 1**

Researcher: How do you use the graph of $y = f^{/}(x)$ to find the turning points of $f(x)$.
L12: *At turning point the derivative is zero, y=0.*
Researcher: *Explain what is meant by 'concavity changing'*
L12: Concavity is changing when the second derivative is negative
Researcher: When asked to sketch graph in 1.3 you found f(2) = -8 and f(0) = -4, explain why ?
L12: These are points on the graph using the two values of x=0 and x=2 which I found in 5.1 for turning points.
Researcher: Where did you get f(x)= $-2x^3+3x^2-4$ which you used to find the points on f(x).
L12: I used the given derivative $f^{/}(x) = -3x^2+6x$.
L12 appeared to be facing challenges resolving the given calculus problem. The first question required him to explain how to use the given graph of the derivative $f^{/}(x)$ to find the turning points of the function f (x) and his response was on a procedure to be followed without any reference to the graph. This is an indication that he was unable to interpret the graph in terms of what information it provided about f(x). This demonstrated a lack of a relational understanding of f(x) and the first derivative.
Secondly, the concept of concavity had been misunderstood by L12. His cancelled solution had $f^{//}(x) > 0$, and the second attempt had $f^{//}(x) \leq 0$. This is evidence that the learner had misconceptions about the meaning of '*concavity changing*' concerning the second derivative. The question was asking for the point of inflection, but the learner appeared to be focused on procedures of solving inequalities without an understanding of the concept in question. The other error was in the solution of $f^{//}(x) \leq 0$, with the final answer from $-6x \leq -6$ is $x \leq 1$ which is incorrect. The correct answer should be $x \geq 1$, after division by -6 on both sides. This error was a result of a flawed knowledge of inequalities which is now creating barriers in the learner's attempts to learn new calculus concepts.
Thirdly, L12 made an error in his attempt to derive the equation of *f(x)* from the equation of first derivative $f^{/}(x) = -3x^2+6x$.The given derivative could be of any function $f(x) = -ax^3 + bx^2 + c$ where c is any constant. The learner demonstrates a lack of understanding of the relationship

between the function and its derivative. In addition, the learner appears not to understand functional notation as he is given $f(2) = 0$, which means (2;0) is a point on the graph of *f(x)*, hence his turning point of $f(2) = -8$ could not be correct if it is the same graph. The learner marks two points (0;-4) and (2;-8) as TPs on the graph, but his graph does not turn at (0;-4). This is evidence that the learner is merely going through procedures without a conceptual understanding of the mathematical concepts involved. The root cause could be teaching and learning methods which emphasise drill and practice at the expense of conceptual knowledge.

Discussion

These common knowledge and misconceptions among the grade 12 students when learning functions and derivatives are discussed under the headings below.

- Conceptual/domain misconception
- Generalisation or transfer errors
- Ignorance of the function rule
- Incomplete application of the sketching method
- Wrong hypothesis and plotting procedure.

The outcome of the sample reported when presenting the views of the grade 12 students on the common knowledge and misconceptions on functions and derivative graph had shown that when assessing the learners' knowledge on functions, an interrelationship do exist between functions and these had resulted to some misconceptions when learning them. Therefore, the underlisted sections were reported because of the finding reported by the researchers.

***Conceptual errors**:* This is a form of misconceptions observed when the learners confused the *x*-intercepts of the graph of the derivative with the *x*-intercepts of the function, reflecting misconception on the geometric meaning of the derivative. This is a conceptual error discovered in the derivative of a function where learners struggled to interpret $y = f(x) = 0$. The *x*-intercepts of $y = f'(x)$ indicate the turning points of the function. Secondly, some learners treated the derivative function as the function *f(x)*, and used the second derivative to find turning points of the function. The other misconception was interpreting the turning points of the graph of the derivative as the turning point of the function, $f(x)$, yet the TP on $f'(x)$ is the point of inflection. This pointed to learner misconceptions on the concept of a derivative of a function.

Wrong hypothesis and plotting procedure

Whenever, there is an evidently lack of understanding of the relationship between the three symbols *f(x)*, $f'(x)$ and $f''(x)$, *as* exposed by statements from learners, such as $f'(x) = -3x^2 + 6x = -6x + 6 = 0$; $x = 1$ where $f'(x)$is treated as $f(x)$ and $f''(x)$ is used as $f'(x)$. A form of wrong hypothesis and lang of understanding on the plotting procedure is discovered. This was what Borasi (1987) referred to as the wrong hypothesis used to learn new concepts as the learner is equating $f'(x)$ and $f''(x)$ in trying to resolve the calculus problem. L16 in figure 4, which introduced *f(x)* from $f'(x) = -3x^2 + 6x$, giving $f(x) = -x^3 + 3x^2$ instead of $f(x) = -x^3 + 3x^2 + c$, demonstrating a lack of understanding of the relationship between *f(x)* and $f'(x)$.

Ignorance of the function rule

This is another form of misconception and error that was committed by L16 which could be regarded as an error of an ignorance of rule restrictions on finding *f(x)* from the derivative $f'(x)$. Hence when required to sketch the graph of $\mathbf{f}(x) = -ax^3 + bx^2 + cx - 4$ where $f(2) = 0$ and $f'(x) = -3x^2 + 6x$, the learners struggles to come up with the correct graph.

Generalisation or transfer errors

The generalisation or transfer errors on *f(x)*, $f'(x)$ and $f''(x)$ and the interrelationship between them. It's a form of error when the learners fail to demonstrate the appropriate level of conceptual understanding of what information is provided by $f(x)$ and $f'(x)$ in $f(x) = -ax^3 + bx^2 + cx - 4;$ $f(2) = 0$ and $f'(x) = -3x^2 + 6x$, as well as how this information is interlinked because they relied on their instrumental understanding of derivatives. These findings confirm Orton's (1983) observations that students may have adequate procedural knowledge of routine differentiation, but lack conceptual understanding of the derivative. Porter and Masingila (2000) also made similar observations on students not having problems with simple procedures of differentiation and finding limits, but struggling when required to demonstrate a deeper understanding of some underlying mathematical notions validating these procedures. Sample data for this research shows examples where L11 in figure 8 and L10 in Figure 7 ignore critical information such as $f(2) = 0$ or marking point f(0) = -4, They are not using this information when sketching the graph or finding the x-coordinates of turning points from $f'(x) = -3x^2 + 6x = 0$, as L10 in Figure 7 did, but could not use these answers to produce the final graph.

Incomplete application of the sketching method

This is a form of error and misconceptions resulted from the learners' incomplete application of rules for finding intercepts and turning points on graphs of functions. This is an evidence of learners' blurred understanding of $f(x)$, $f'(x)$ and $f''(x)$, their interrelationship and the geometric meaning of each.

The major source of this error could lie on the teaching and learning approaches which appear to have emphasised the acquisition of procedural knowledge at the expense of conceptual understanding (Aspinwall & Miller 1997; Bezuidenhout, 2001; Davis &Vinner, 1986; Morris, 1999; Toh, 2007). This research also confirmed the findings by Carlson (Carlson et al., 2003), that in the process of solving calculus problems, students failed to utilise all the given information, opting to selectively utilise only part of the information which they considered relevant. They ignored other parts information that would be indispensable in the successful resolution of the problem.

Conclusions

In this study, it could be clearly seen that some grade 12 students encountered some misconception due to their common knowledge on functions and derivatives as reported in the finding and discussion sections. Despite the common knowledge and misconceptions reported, the topics on calculus cannot be removed from the grade 12 curriculum rather the Mathematics teachers teaching the students should strategized some modern methods of teaching functions and derivatives graph to improve the students' performance in functions and derivatives. To reduce the misconceptions such as Conceptual misconception, generalisation or transfer errors, ignorance of the function rule, incomplete application of the sketching method, wrong hypothesis and plotting procedure among many others. There should be a thorough in-service training among the practising teachers. Because it is generally believed that most of the students find it difficult to calculate function and derivative without committing one error and others. This result is consistent with that of some other researchers who worked on functions of a graph and derivatives in mathematics education as earlier discussed.

References

Atar, B. Ş. Y., & Aykutlu, I. (2023). High School Students' User Skills Concerning Force and Motion Graphs. *Gazi Üniversitesi Gazi Eğitim Fakültesi Dergisi*, *43*(1), 211-242. https://doi.org/10.17152/gefad.1205369

Asiala, M., Cottrill, J., Dubinsky, E., & Schwingendorf, K. E. (1997). The development of students' graphical understanding of the derivative. *The Journal of Mathematical Behavior,* 16(4), 399-431. https://doi.org/10.1016/S0732-3123(97)90015-8

Aspinwall, L., & Miller, D. (1997). Contrasting cases of calculus students' understanding of derivative graphs . *Mathematical Thinking and Learning*, 12(2), 152-176.

Bezuidenhout, J. (2001). Limits and Continuity: Some conceptions of first year students. *International Journal of Mathematics Education in Science and Technology,* 32(4), 487-500. https://doi.org/10.1080/00207390010022590

Bursal, M., & Yetis, S. (2020). Middle School Students' Graph Skills and Affective States about Graphs. *International Journal of Research in Education and Science*, 6(4), 692-704. https://doi.org/10.46328/ijres.v6i4.1136

Bragdon, D., Pandiscio, E., & Speer, N.(2019). University students' graph interpretation and comprehension abilities. *Investigations in Mathematics Learning*, *11*(4), 275-290. https://doi.org/10.1080/19477503.2018.1480862

Borji, V., Alamolhodaei, H., & Radmehr, F. (2018). Application of the APOS-ACE theory to improve students' graphical understanding of derivative. *EURASIA journal of mathematics, science and technology education,* 14(7), 2947-2967.

Darren M. (2019). The trouble with graph. https://www.cambridgemaths.org/blogs/the-trouble-with-graphs/

Bursal, M., & Polat, F. (2020). Middle School Students' Line Graph Skills and Affective States about Common Graph Types Used in Science Courses. *International Journal of Education in Mathematics, Science and Technology*, 8(4), 290-303.

Carlson, M., Persson, J .&Smith ,N. (2003). Developing and connecting students' notions of rate of change and accumulation. The fundamental theorem of calculus . In Proceedings of the 2003 *Meeting of the International Group for Psychology of Mathematics Education-North America*, (2), 165-172.

De Ancheta, C. M. (2022). An Error Analysis of Students' Misconceptions and Skill Deficits in Pre-Calculus Subjects.

Davis, R B., &Vinner, S. (1986). The notion of Limit: Some seemingly unavoidable misconceptions stages. *Journal of Mathematical Behaviour,* 5(3), 281-303.

Fasinu, V. G. (2024). An Interactive Approach of Teaching and Learning Prerequisite Mathematics Course and Core Engineering Mathematics Course During COVID-19 Pandemic. *Interdisciplinary Journal of Education, 7(2), 184-202.*

Hadjidemetriou, C., & Williams, J. (2002). Children's graphical conceptions. *Research in Mathematics Education*, 4(1), 69-87.

İlkörücü, Ş., & Broutın, M. S. T. (2022). Evaluation of Pre-Service Science Teachers' Visual and Cognitive Constructions of A Line Graph. *Uludağ Üniversitesi Eğitim Fakültesi Dergisi*, 35(3), 647-66.

Jameson G. Machaba, M. F. & Fasinu, V. G. (2025). Grade 12 Students' Misconceptions when Modelling their Calculus Ideas into their Learning of an Optimization: A Real-Life Problem. *Unpublished Paper*.

Jameson, G., Machaba, M. F., & Fasinu, V. G. (2024). Misconceptions and Errors Among Grade 12 Students When Learning Differentiation Rules: A Case Study. *Mathematics Education Journal*, 8(2), 221-243.

Jones, S. R., & Watson, K. L. (2018). Recommendations for a "target understanding" of the derivative concept for first-semester calculus teaching and learning. *International journal of research in undergraduate mathematics education*, (4), 199-227.

Körner, C. (2005). Concepts and misconceptions in comprehension of hierarchical graphs. *Learning and Instruction*, 15(4), 281-296. https://doi.org/10.1016/j.learninstruc.2005.07.003

Kranda, S., & Akpınar, M. (2018). Defining of Graphical Literacy Levels of 7 th Grade Students' in Social Studies Course. *Journal of Kirsehir Education Faculty*, 19(3).

Morris, A. (1999).Developing concepts of mathematical structure : pre-arithmetic reasoning versus extended arithmetic reasoning. *Focus on Learning Problems in Mathematics*, 21(1), 44-71.

Makonye, J. P., & Luneta, K. (2014). Mathematical errors in differential calculus tasks in the Senior School Certificate Examinations in South Africa. *Education as Change*, 18(1), *119-36*. https://doi.org/10.1080/16823206.2013.847014

Ngulube, B., & Ogbonnaya, I. C. (2023). Applying Bandura's Theory of Self-Efficacy to assess the perceptions of Pre-service Economics teachers concerning teaching using graphs. *International Journal of Research in Business and Social Science* (2147-4478), 12(8), 517-525.

Orton, A. (1983). Students understanding of differentiation. *Educational studies in mathematics*, (14), 165177. https://doi.org/10.1007/BF00410540

Ozaltun-Celik, A. (2021). A calculus student's understanding of graphical approach to the derivative through quantitative reasoning. *LUMAT: International Journal on Math, Science, and Technology Education*, 9(1), 892–916. https://doi.org/10.31129/LUMAT.9.1.1663

Ozmen, Z. M., Guven, B., & Kurak, Y. (2020). Determining the graphical literacy levels of the 8th grade students. *Eurasian Journal of Educational Research*, 20(86), 269-292. https://doi.org/10.14689/ejer.2020.86.13

Özgen, K., Aygün, N., & Hanazay, H. (2017). High School Students' Graphing Skills of Trigonometric Functions. *Necatibey Faculty of Education Electronic Journal of Science & Mathematics Education*, 11(2).

Porter, M.K., & Masingila, J.O. (2000). Students learning calculus. *Educational Studies in Mathematics* (42), 165–177. https://link.springer.com/content/pdf/10.1023/A:1004166811047.pdf

Rodriguez, J. M. G., & Jones, S. R. (2021). Identifying Graphical Forms Used by Students in Creating and Interpreting Graphs. In *North American Chapter of the International Group for the Psychology of Mathematics Education*.

Sahin, Z., Yenmez, A. A., & Erbas, A. K. (2015). Relational understanding of the derivative concept through mathematical modeling: A case study. *Eurasia Journal of Mathematics, Science and Technology Education*, 11(1), 177-188.

Syarifuddin, A., & Sari, A. F. (2021, April). Misconceptions of prospective Mathematics teacher on graphing function. In *Journal of Physics: Conference Series* (Vol. 1869, No. 1, 012115). IOP Publishing.

Ural, A. (2013). Ninth Graders' Understanding of The Concept of Function Within a Graphical Representation. *International Journal of Multidisciplinary Thought*, 3(4), 401-415.

Zandieh, M. (2000). A theoretical framework for analyzing student understanding of the concept of derivative. In E. Dubinsky, A. H. Schoenfeld, & J. Kaput (Eds.). *Research in Collegiate* Mathematics Education (Vol. IV, pp. 103–127). Providence, RI: American Mathematical Society. https://doi.org/10.1090/cbmath/008/06

Zehra, A., & Abbasi, S. J. (2019, October). Misconceptions of derivative and integration techniques among engineering students: a case study. *In Journal of Physics: Conference* Series (Vol. 1320, No. 1, p. 012010). IOP Publishing.

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РОЗУМІННЯ ТА ХИБНІ УЯВЛЕННЯ УЧНІВ 12-ГО КЛАСУ ПІД ЧАС ВИВЧЕННЯ ГРАФІКІВ ФУНКЦІЙ І ПОХІДНИХ У МАТЕМАТИЧНОМУ АНАЛІЗІ

Анотація. Математичний аналіз ґрунтується на розумінні графічного подання функцій та їхніх похідних, що залишається важливим складником математичної освіти. Водночас у процесі вивчення цих тем в учнів формуються численні хибні уявлення, які можуть ускладнювати подальше опанування математичного аналізу. Одним із поширених помилкових уявлень є трактування нахилу прямої, проведеної через певну точку, як безпосереднього вираження похідної функції в цій точці. Таке розуміння не враховує того, що дотична є границею січних прямих у процесі їх наближення до точки дотику. Як наслідок, поняття похідної та сфери її застосування інтерпретуються надмірно спрощено.

Ще одним поширеним непорозумінням є сприйняття похідної як простого відношення *dy* до *dx* замість усвідомлення її як границі середньої швидкості зміни функції. Через це учні часто виконують диференціювання механічно, без належного розуміння його теоретичних засад. У результаті вони припускаються помилок під час визначення внутрішніх і зовнішніх функцій, а також не розпізнають композицію функцій. Це, своєю чергою, спричиняє труднощі у застосуванні графічного методу обчислення похідних.

Результати дослідження дали змогу виявити типові хибні уявлення та помилкові припущення, що виникають у процесі засвоєння нових математичних понять. До них належать концептуальні хибні уявлення, концептуальні помилки, помилки узагальнення та перенесення знань, ігнорування обмежень правил, а також недостатнє застосування правил. Для досягнення мети дослідження було використано якісний метод із залученням учнів 12-го класу південноафриканської середньої школи, які вивчають математику.

Ключові слова: хибні уявлення; знання; математичний аналіз; функція; графік.